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Comments on the paper "The Mexican Hat Wavelet Family. Application to point source detection in CMB maps" by J. González-Nuevo et. al (astro-ph/0604376)

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Abstract. The arguments presented by González-Nuevo et al. (2006) in favour of the *Mexican Hat Wavelet Family* (MHWF) are critically discussed here. These authors allege the *optimal* properties of this new class of filters in the detection of point sources embedded in a noise background but their claim is not based upon a solid mathematical foundation and proof.

Key words. Methods: data analysis – Methods: statistical

1. Introduction

The detection of (extragalactic) point sources embedded in a noise background is a critical issue in the analysis of the experimental CMB maps. Actually, this kind of problem is not only restricted to Cosmology. In fact, this old dated and extensively studied question is of relevance in many scientific and engineering applications. Various approaches have been proposed. Among these, one of the most popular is the *matched filter* (MF) technique. There are good reasons to choose it (see Kay 1998):

– In the case of a background due to a Gaussian stochastic process (a rather common assumption), MF is *optimal* in the Neyman-Pearson sense. This means that, for a given probability of *false alarm* (i.e., the probability of a spurious detection), it provides the best probability of detection;

– If the Gaussian assumption is relaxed, MF still maximizes the *signal-to-noise ratio* (SNR) at the output of *any* linear filter. In other words, among the linear filters, MF provides the greatest enhancement of the magnitude of the source relative to the background;

– As proved by its extensive and successful use in very different contexts over many years, MF is a quite robust tool.

In spite of these remarkable characteristics, in the CMB literature a long series of papers were published where the use of alternative techniques (e.g., *pseudo-filters*, *Mexican Hat Wavelet filters* and *biparametric-scale adaptive filters*) is advocated and claimed to provide superior results (e.g., see Sanz et al. 2001; Barreiro et al. 2003; López-Caniego et al. 2005). In another series of papers (Vio et al. 2002, 2004; Vio & Andreani 2005a,b) it was shown that such claims were only the consequence of a incorrect interpretation of the results and/or of the incorrect application of the validity conditions of some equations. Of course, this does not mean that MF necessarily has to be chosen in every point source detection problems. However, given its excellent properties, the use of alternative approaches must be motivated on the basis of well-grounded theoretical arguments. Non-standard statistical tools are indicated only in situations of real and sensible improvements of the results. New techniques that do not fulfill these requirements should be introduced with care: they prevent the comparison with the results obtained in other works, may lead people to use not well tested methodologies and end up in unreliable results.

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Now, González-Nuevo et al. (2006) (*Go06*) seem do not agree on this commonsense rule since they propose a new class of filters, say the *Mexican Hat Wavelet Family* (MHWF), that is derived from the iterated application of the Laplacian operator to the *Mexican Hat Wavelet* (MHW). However, they do not provide any formalized argument. Their claims are supported only on numerical simulations. This is certainly not a safe way to proceed since, contrary to a rigorous theoretical treatment, a set of numerical experiments is not sufficient to characterize the statistical properties of a given methodology.

2. The Mexican Hat Wavelet Filter vs. the Matched Filter

Go06 start from the consideration that, as written in their paper, *" the Mexican Hat Wavelet is a very useful and powerful tool for point source detection due to the following reasons:*

1. *It has an analytical form that is very convenient when making calculations and that allows us to implement fast algorithms.*

2. *It is well suited for the detection of Gaussian structure because it is obtained by applying the Laplacian operator to the Gaussian function.*

3. *It amplifies the point sources with respect to the noise. Moreover, by changing the scale of the Mexican Hat it is possible to control the amplification until an optimum value is achieved.*

4. *Besides, to obtain the optimal amplification it is not necessary to assume anything about the noise. In Vielva et al. (2001), it was shown that the optimal scale can be easily obtained by means of a simple procedure for any given image. Therefore the Mexican Wavelet is a very robust tool.".*

Unfortunately, all of these points are irrelevant and/or false:

1. This property is overestimated. The computation of MF does not present particular difficulties (e.g., see Vio et al. 2002). Moreover MHW and MF are linear filters, hence both of them can be implemented in very efficient algorithms;

2. It is not clear on the basis of which argument authors make this claim. Is there any theoretical explanation for it? Moreover, if the conviction of authors is correct, why in the only analytical example presented in their paper (i.e., Gaussian source plus white-noise background) the filter that provides the best source amplification is not MHF but a Gaussian filter that, in this case, corresponds to MF?

3. Whatever filter that smoothes out the frequencies characteristic of the background amplifies the point source with respect to the noise. But the best amplification is provided by MF not by MHW (see above). This is true even in the case of the scale-adaptive version of MHF. In this respect, it is necessary to stress a point that in González-Nuevo et al. (2006) is not adequately highlighted. In general, the performances of MHW are by far inferior to those of MF (e.g., see Vio et al. 2002, 2004). To make the scale R a free parameter in the analytical form of MHF is only an attempt to improve a performance otherwise unsatisfactory. The situation is similar to that of the fit of a set of data with an inadequate model. If results are not good, the simplest solution is to add one more free parameter to it. Why one should choose MHW with a free parameter to be optimized when the only result that can be expected is a filter at the best suboptimal with respect to MF?

4. As written above, among all the linear filters, MF provides an optimal source amplification independently from the nature of the noise. For this reason, it is not clear on which basis *Go06* claim that this property belong to MHW.

More in general, *Go06* plead that, given their ability to explore a signal at different scales, the wavelets represent an effective tool in detection problems. However, it is not clear why this property should be useful in a context where all the sources have the same shape that, in addition, is known in advance. Why wavelets should provide superior results than the techniques, as MF, that make direct use of the shape of the sources? If a reason is available, this should be formally proved.

As last comment, we stress that MF can be easily adapted when dealing with sources with the same shape but different amplitude. It it not difficult to prove that MF remains identical to that corresponding to a single amplitude. Only the threshold determining the *probability of false alarm* (hence also the probability of detection), is modified. If the distribution of the amplitudes is known in advance, such threshold can be fixed through a Bayesian approach or, especially in the case of non-Gaussian background, through numerical simulations.

3. The Mexican Hat Wavelet Family

By themselves, all the above points should be sufficient to question the real usefulness of a generalization of MHW. However, let's assume for a moment that MHW has really optimal properties: which is the theoretical reason to construct a family of filters through the iterated application of the Laplacian operator to MHW? Why should such a procedure improve the performances of MHW? Quite surprisingly, *Go06* do not provide any theoretical argument. The situation appears even more bizarre if one considers again the only analytical example presented in their paper

(Gaussian source plus white-noise background). There, the source amplification worsens for increasing values of the order n. The *best* solution is given by $n = 0$. Since this case corresponds to the Gaussian filter, i.e. to MF, hopefully *Go06* do not consider MF belonging to MHWF!

The "optimal" performances of MWHF is supported only on the basis of the CMB numerical experiments carried out by authors. We stress that this is insufficient to drawn reliable conclusion about MHWF also because no whatever comparison is made with other filters and specifically with MF.

4. Conclusions

In this brief note the procedure suggested by (González-Nuevo et al. 2006) for the detection of point sources in CMB maps is criticized. The introduction of a new class of filters, the *Mexican Hat Wavelet Family*, is given without providing any theoretical argument about their real properties and usefulness. We stress that any new proposed statistical methodology cannot be validated only on the basis of some numerical experiments, especially if no comparison with classic and well tested techniques is made.

The risk of an uncontrolled proliferation of algorithms (and papers) whose reliability is at least dubious must be avoided and we strongly hope that a more careful check on the scientific foundations will be operated in the future.

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